Coulomb corrections to the three-body correlation function in high-energy heavy ion reactions

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Abstract

Starting from an asymptotically correct three-body Coulomb wave-function, we determine the effect of Coulomb final state interaction on the three-particle Bose-Einstein correlation function of similarly charged particles. We numerically estimate that the Riverside approximation is not precise enough to determine the three-body Coulomb correction factor in the correlation function, if the characteristic HBT radius parameter is 5 - 10 fm, which is the range of interest in high-energy heavy ion physics.

Key words: three-body Coulomb, correlations, high-energy, heavy-ion

1 Introduction

One of the most important tasks of high energy heavy-ion studies is to prove the existence of the elusive quark-gluon plasma and to study the properties of this predicted new state of matter[1]. Hanbury-Brown Twiss (HBT) interferometry[2–6] of identical particles has become an important tool as it can be used to measure the evolving geometry of the interaction region. The quantitative interpretation of the HBT-results depends however critically on

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the understanding of the rôle of the Coulomb interaction between the detected particles as well as on the influence exerted on the selected system by the remaining particles[7–9]. In the attempts to create the quark-gluon plasma we use higher and higher energies and larger and larger colliding nuclei. In these collisions the multiplicity of particles gets very large and the importance of the HBT-study of more than two particles will grow. Recently, much work [10–12] has been devoted to what can be learned from the system of three charged pions, however without paying sufficient attention on how the Coulomb interaction might change the predictions. In the few existing experimental studies on the HBT effect in the three-charged pion system, see [13] and references given there, Coulomb effects have been taken into account in the so-called Riverside approximation [14], or sometimes in more elaborated versions of it[15]. However, these schematic conceptual ansaetze are acceptable only because a more accurate multiparticle Coulomb interaction treatment has not been available.

In this Letter we propose a proper treatment of the Coulomb interaction between three charged particles. It is based on non-relativistic three-particle scattering theory, assuming that their velocities relative to each other are non-relativistic which, however, is just the kinematic situation where the HBT study of three particles is of interest. We, moreover, show that the Riverside approximation follows as a special case when the source of particle creation becomes very small, in the same way as the Gamow penetration factor is the limit for the Coulomb correction in the two-particle correlation function. An estimate of the magnitude of the corrections for source sizes to be expected at RHIC and LHC is presented using current data from NA44 experiment on S-Pb collisions at CERN SPS.

2 Coulomb corrections to three-body correlations

2.1 Three-charged particle wave function

In order to treat correctly the Coulomb corrections to the three-body correlation function knowledge of the three-particle Coulomb wave function is required. As mentioned in the Introduction, we restrict ourselves to the case that the transverse momenta of the three particles in the final state are small enough to make a nonrelativistic approach applicable. Hence the problem consists in finding the solution of the three-charged particle Schrödinger equation when all three particles are in the continuum.

Consider three distinguishable particles with masses m_i and charges e_i, $i = 1, 2, 3$. Let $\mathbf{x}_i$ and $\mathbf{k}_i$ denote the coordinate and momentum (three-)vectors,

respectively, of particle i. From these we construct in the usual manner the relative coordinate $\mathbf{r}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ and the relative momentum $\mathbf{k}_{ij} = (m_j\mathbf{k}_i - m_i\mathbf{k}_j)/(m_i + m_j)$ between particles i and j, the corresponding reduced mass being $\mu_{ij} = m_i m_j/(m_i + m_j)$.

The three-particle Schrödinger equation reads

$$\left\{ H_0 + \sum_{i<j=1}^{3} V_{ij} - E \right\} \Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = 0, \tag{1}$$

where

$$E = \sum_{i=1}^{3} \frac{k_i^2}{2m_i} > 0 \tag{2}$$

is the total kinetic energy for three particles in the continuum. H_0 is the three-free particle Hamilton operator and

$$V_{ij}(\mathbf{r}_{ij}) = V^S_{ij}(\mathbf{r}_{ij}) + V^C_{ij}(\mathbf{r}_{ij}) \tag{3}$$

the interaction potential between particles i and j, consisting of a sum of a strong short-range (V^S_{ij}) plus the long-range Coulomb interaction ($V^C_{ij}(\mathbf{r}_{ij}) = e_i e_j/r_{ij}$).

The exact numerical solution of the Schrödinger equation (1) for $E > 0$ is beyond present means, partly for principal and partly for practical reasons. For a brief discussion of the related difficulties see [16]. But, at least in all asymptotic regions of the three-particle configuration space is the form of its solution nowadays known analytically [17,18]. It is simplest in the asymptotic region usually denoted by Ω_0 and characterized by the fact that - roughly speaking - all three interparticle distances become uniformly large, i.e., $r_{12}, r_{23}, r_{31} \to \infty$ (for a precise definition of the various asymptotic regions see [18]).

A convenient and widely used representation of a three-charged particle wave function which coincides in Ω_0 with the correct asymptotic wave function is given by [19,20]

$$\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \overset{\Omega_0}{\approx} \psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{12}) \psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{23}) \psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{31}). \tag{4}$$

Here, $\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij})$ is the continuum solution of the two-body Coulomb Schrödinger equation

$$\left\{-\frac{\Delta_{\mathbf{r}_{ij}}}{2\mu_{ij}} + V^C_{ij}(\mathbf{r}_{ij}) - \frac{k^2_{ij}}{2\mu_{ij}}\right\} \psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij}) = 0, \tag{5}$$

describing the relative motion of the two particles i and j with energy $k^2_{ij}/2\mu_{ij}$. The explicit solution of (5) is known,

$$\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij}) = N_{ij} e^{i\mathbf{k}_{ij}\cdot\mathbf{r}_{ij}} F[-i\eta_{ij}, 1; i(k_{ij}r_{ij} - \mathbf{k}_{ij}\cdot\mathbf{r}_{ij})], \tag{6}$$

with $N_{ij} = e^{-\pi\eta_{ij}/2}\,\Gamma(1+i\eta_{ij})$, and $\eta_{ij} = \frac{e_i e_j \mu_{ij}}{k_{ij}}$ being the appropriate Coulomb (Sommerfeld) parameter. $F[a, b; x]$ is the confluent hypergeometric function and $\Gamma(x)$ the Gamma function. When writing the asymptotic solution of (1) in the form (4), use has been made of the fact that in Ω_0, the short-range part V^S_{ij} of the two-particle interactions can be neglected and, hence, only the Coulomb potentials survive.

Let us add a few comments.
(i) It is to be emphasized that in using a wave function of the type (4), the three-body system is considered as a sum of three noninteracting two-body systems (each on its two-body energy shell). That is, neither correlations between the motions of the three particle pairs nor off-the-two-body-shell effects are included, which obviously can be true at most for asymptotic particle separations (in fact, it holds true in, and only in, Ω_0).
(ii) The wave function (4) provides a well-defined prescription of how to go to arbitrary, in particular small, values of the relative coordinates. Though, it ceases to be solution of the Schrödinger equation (1) for non-asymptotic values of the relative coordinates; nevertheless, it has proved to be rather successful in describing $(e, 2e)$ processes in atomic physics. But it should be kept in mind that the extrapolation out of the region Ω_0 implied by (4) is highly non-unique. In fact, several other ansaetze which, of course, coincide asymptotically in Ω_0 with (4) have been, and are still being, developed.
(iii) We mention that ansaetze for three-charged particle wave functions have been proposed which are correct in all asymptotic regions (see, e.g., [18]). As is to be expected, they contain correlations between the motions of the three particles. However, their form is rather more complicated and, thus, will not be used for the present investigation. For some more details and for further references see [21].

Hence, in the present investigation we assume the three-charged particle wave function to be given in all of three-body configuration space as

$$\tilde{\Psi}^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \;:=\; \mathcal{N} \psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{12}) \psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{23}) \psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{31}). \tag{7}$$

Here, $\mathcal{N}$ is an (undetermined) overall normalization constant.

2.2 Specialization to three identical bosons

Given a general three-particle wave function $\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ it is a simple task to specialize to various interesting situations. As a particular application we treat explicitly only the case of three identical bosons with unit charges of magnitude $|e|$ and mass m. The properly symmetrized wave function is

$$\begin{aligned}\Psi^{(+)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) = \frac{1}{\sqrt{6}}\Big\{&\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)+\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_3,\mathbf{x}_2)\\ &+\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_2,\mathbf{x}_1,\mathbf{x}_3)+\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_1)\\ &+\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_3,\mathbf{x}_1,\mathbf{x}_2)+\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_3,\mathbf{x}_2,\mathbf{x}_1)\Big\}. \end{aligned} \tag{8}$$

By choosing for $\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$ the - at least asymptotically in Ω_0 correct - ansatz

$$\Psi^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) = \tilde{\Psi}^{(+)}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3), \tag{9}$$

we find for the approximate symmetrized wave function the form

$$\begin{aligned}\tilde{\Psi}^{(+)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) = \frac{\mathcal{N}}{\sqrt{6}}\Big\{&\psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{12})\psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{23})\psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{31})\\ &+\psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{13})\psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{32})\psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{21})\\ &+\psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{21})\psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{13})\psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{32})\\ &+\psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{23})\psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{31})\psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{12})\\ &+\psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{31})\psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{12})\psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{23})\\ &+\psi^{C(+)}_{\mathbf{k}_{12}}(\mathbf{r}_{32})\psi^{C(+)}_{\mathbf{k}_{23}}(\mathbf{r}_{21})\psi^{C(+)}_{\mathbf{k}_{31}}(\mathbf{r}_{13})\Big\}. \end{aligned} \tag{10}$$

The case that all three particles are uncharged follows from (10) by substituting for the Coulomb wave functions $\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij})$ the plane waves $e^{i\mathbf{k}_{ij}\cdot\mathbf{r}_{ij}}$. We denote the corresponding symmetrized three-uncharged particle wave function by $\tilde{\Psi}^{(0)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)$:

$$\begin{aligned}\tilde{\Psi}^{(0)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3) := \frac{\mathcal{N}_0}{\sqrt{6}}\Big\{&e^{i\mathbf{k}_{12}\cdot\mathbf{r}_{12}}e^{i\mathbf{k}_{23}\cdot\mathbf{r}_{23}}e^{i\mathbf{k}_{31}\cdot\mathbf{r}_{31}}+e^{i\mathbf{k}_{12}\cdot\mathbf{r}_{13}}e^{i\mathbf{k}_{23}\cdot\mathbf{r}_{32}}e^{i\mathbf{k}_{31}\cdot\mathbf{r}_{21}}\\ &+e^{i\mathbf{k}_{12}\cdot\mathbf{r}_{21}}e^{i\mathbf{k}_{23}\cdot\mathbf{r}_{13}}e^{i\mathbf{k}_{31}\cdot\mathbf{r}_{32}}+e^{i\mathbf{k}_{12}\cdot\mathbf{r}_{23}}e^{i\mathbf{k}_{23}\cdot\mathbf{r}_{31}}e^{i\mathbf{k}_{31}\cdot\mathbf{r}_{12}}\\ &+e^{i\mathbf{k}_{12}\cdot\mathbf{r}_{31}}e^{i\mathbf{k}_{23}\cdot\mathbf{r}_{12}}e^{i\mathbf{k}_{31}\cdot\mathbf{r}_{23}}+e^{i\mathbf{k}_{12}\cdot\mathbf{r}_{32}}e^{i\mathbf{k}_{23}\cdot\mathbf{r}_{21}}e^{i\mathbf{k}_{31}\cdot\mathbf{r}_{13}}\Big\}, \end{aligned} \tag{11}$$

where $\mathcal{N}_0$ is the appropriate normalisation constant.

A simple approximation to (10) which incorporates at least part of the Coulomb effects is obtained by substituting for each of the two-particle Coulomb wave functions $\psi^{C(+)}_{\mathbf{k}_{ij}}(\mathbf{r}_{ij})$ the term $e^{i\mathbf{k}_{ij}\cdot\mathbf{r}_{ij}}N_{ij}$, i.e., by neglecting the hypergeometric function part in the exact solution (6) (this is justified if, for all pair of indices $(ij) = (12), (23), (31)$, the arguments $(k_{ij}r_{ij} - \mathbf{k}_{ij}\cdot\mathbf{r}_{ij})$ of the hypergeometric functions are sufficiently small, as it happens, e.g., for small relative distances). This leads to

$$\tilde{\Psi}^{(+)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \approx N_{12}N_{23}N_{31}\Psi^{(0)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3). \tag{12}$$

For the probability density we arrive at

$$\left|\tilde{\Psi}^{(+)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)\right|^2 \approx G_{12}G_{23}G_{31}\left|\tilde{\Psi}^{(0)\mathcal{S}}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)\right|^2, \tag{13}$$

where the Gamov factors $G_{ij} \equiv |N_{ij}|^2$ have been introduced. This is nothing but the well known Riverside approximation which consequently finds some theoretical justification within the framework of the exact nonrelativistic three-body scattering theory as a kind of lowest-order approximation. But this justification is rather weak. For, in order to "derive" the Riverside approximation, the wave function (7) or (10) is used in the region $r_{12}, r_{23}, r_{31} \approx 0$ where it is certainly a much poorer approximation to the exact solution of (1) than at the larger values of the relative particle separations as applied in the following. Thus, a better justified expression is obtained by using the wave function (10). Nevertheless, also in that case an overall factor $N_{12}N_{23}N_{31}$ can be extracted from the r.h.s. of (10), which for the probability density yields again a factor $G_{12}G_{23}G_{31}$. Note that in the context of three-body scattering theory we do not find any support for the Coulomb correction factors proposed in [15].

2.3 Application to high-energy heavy-ion collisions

The correlation function measuring the enhanced probability for emission of three identical Bose-particles is given by:

$$C_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{N_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{N_1(\mathbf{k}_1)\, N_1(\mathbf{k}_2)\, N_1(\mathbf{k}_3)} \tag{14}$$

where $\mathbf{k}_i$ is the three-momentum of particle i. The three-particle momentum distribution is $N_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ and the single-particle momentum distribution

is denoted by $N_i(\mathbf{k}_i)$. This correlation function is presently, due to meagre statistics, only measured as a function of the Lorentz invariant Q_3, defined by the relation

$$Q_3^2 = k_{12}^2 + k_{23}^2 + k_{31}^2 \tag{15}$$

where $k_{ij} = k_i - k_j$. In these equations is k_i the four-momentum vector defined by the three-momentum $\mathbf{k}_i$ and the mass of particle i, m_i.

We can now calculate the Coulomb effect on this three-particle correlation function using

$$K_{Coulomb}(Q_3) = \frac{\int d^3\mathbf{x}_1 \rho(\mathbf{x}_1) d^3\mathbf{x}_2 \rho(\mathbf{x}_2) d^3\mathbf{x}_3 \rho(\mathbf{x}_3) \left| \tilde{\Psi}^{(+)S}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \right|^2}{\int d^3\mathbf{x}_1 \rho(\mathbf{x}_1) d^3\mathbf{x}_2 \rho(\mathbf{x}_2) d^3\mathbf{x}_3 \rho(\mathbf{x}_3) \left| \tilde{\Psi}^{(0)S}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \right|^2}, \tag{16}$$

where $\rho(\mathbf{x_i})$ is the density distribution of the source for particle i, taken as a Gaussian distribution of width R in all three spatial directions, $\rho(\mathbf{x}) = \frac{1}{(2\pi R)^3} \exp[-(\frac{\mathbf{x}^2}{2R^2})]$. This formulation makes it possible to estimate the Coulomb effect as a function of the radius parameter R on the three-particle correlation function, and to compare this estimate with that calculated by means of the Riverside approximation through $K^{(Rs)}_{Coulomb}(Q_3) = G_{12}G_{23}G_{31}$.

To this purpose we use the NA44 data sample of three pion events produced in S-Pb collisions at CERN [22]. The pions of this sample are identified and well separated charged particles. All magnitudes of relative momenta $|\mathbf{k_{ij}}|$ are larger than 5 MeV due to experimental constraints and Q_3 is larger than 15 MeV.

We have calculated the Coulomb correction, i.e. $K^{-1}_{Coulomb}(Q_3)$, for radius values R=5 and 10 fm, i.e. the range of interest for high energy heavy-ion physics, and compared the result to the Riverside approximation, see Figure 1. For a detailed discussion of these data see the NA44 publication [22].

We find that already for a source of radius 5 fm we need to take the detailed calculation into account as the difference to the Riverside approximation is around 5-10%, furthermore the inset clarifies that this difference is Q_3-dependent, resulting in significant changes in any parameter extracted to characterize the correlation function. For a larger source radius, 10 fm, the difference between the estimates is even more pronounced. We have checked that in the limit $R \to 0$ we recover the Riverside approximation.

In Figure 2 we show the result of a corresponding calculation for the two-pion case [23], illustrating that a finite size of the source reduces the Coulomb effect, as estimated by the Gamow penetration factor, much in the same way

as it reduces the Coulomb effect in the three-pion case from the Riverside approximation.

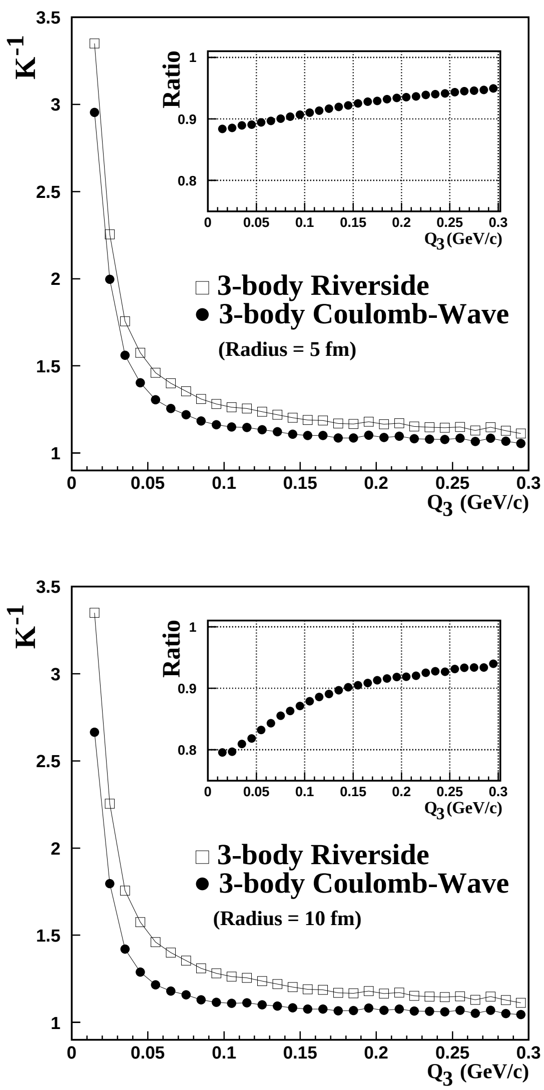

Fig. 1. The figures show the three-pion Coulomb correction factor $K^{-1}_{Coulomb}(Q_3)$ as well as the Riverside approximation. In the upper panel, the input radius was 5 fm, while in the lower, it was 10 fm. The insets display the ratio Coulomb wavefunction integration to the Riverside approximation. Lines are shown to guide the eye.

3 Discussion: Possible areas of future application

Finally we highlight some of the physical problems in high-energy physics where the above results may have a future application or where some measurements already indicate the possible influence of the many-body Coulomb

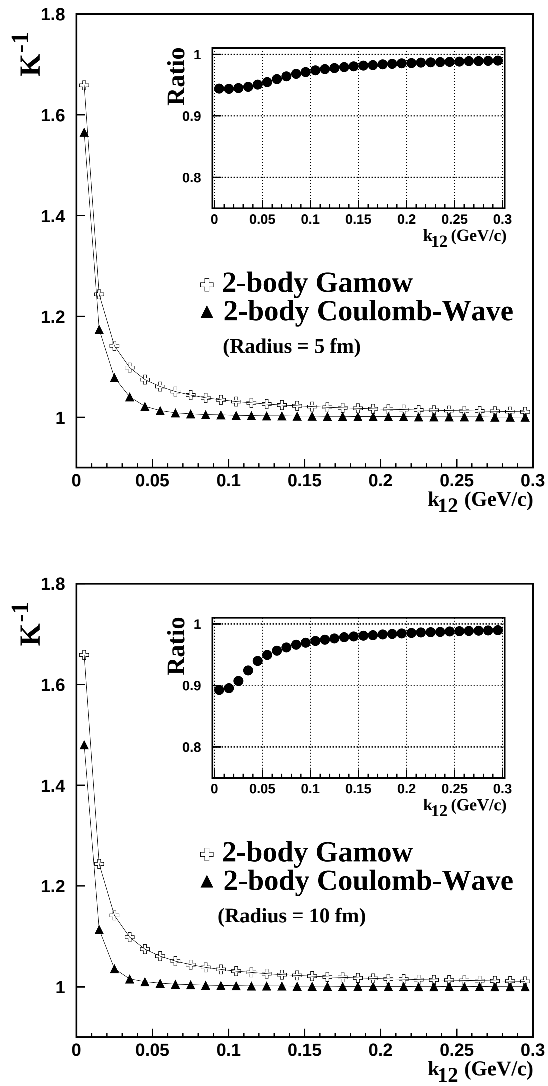

Fig. 2. The figures show the two-pion Coulomb correction factor $K^{-1}_{Coulomb}(k_{12})$ as well as the Gamow approximation. In the upper panel, the input radius was 5 fm, while in the lower, it was 10 fm. The insets display the ratio Coulomb wavefunction integration to the Gamow approximation. Lines are shown to guide the eye.

distortions on the observables.

1) In high-energy heavy ion physics nuclei are collided, that carry a large net positive charge and some of this charge may remain till the end of the reaction in the central collision zone. Such a residual positive charge is expected to distort the single-particle spectra of positively charged pions in different manner than that of the single-particle spectra of negatively charged pions. Namely, negatively charged pions are attracted to lower momenta, while positively charged pions are repelled to higher values of the momenta. This effect was seen in the NA44 data in Ref.[24] and modeled in Ref. [9]. This problem

has a subtle three-body aspect, namely the correlations between the positively and the negatively charged pions induced by their Coulomb interaction which is also of long range, hence in principle cannot be neglected. In this case, we have a charge distribution $(Z, +, -)$ for which our wave function ansatz (7) can be utilized without the need of further symmetrization to determine more precisely the magnitude of the residual charge effects on the $N_1^{(-)}(\mathbf{k})/N_1^{(+)}(\mathbf{k})$ ratio of particle spectra.

2) If a residual net positive charge plays a rôle on the distortion of charged particle spectra, its effects must also be seen on the like-charged two-particle correlation functions. For example, the two-particle intensity correlation functions are symmetric in the absence of a central residual charge, $C^{(++)} = C^{(--)}$, however, any net residual charge breaks the charge conjugation symmetry of the system, $C_Z^{(++)} \neq C_Z^{(--)}$. This question could be properly studied also in the framework of a Coulomb three-body problem with charges $(Z, +, +)$ and $(Z, -, -)$ with the two-body symmetrization of the Coulomb relative wave-functions of the two identical bosons in this system .

3) The two-particle unlike-sign correlation function is also distorted by the Coulomb-effects of the residual charge. If we order the observed particles (pions), e.g., according to the magnitude of their momenta, and fix this order during the determination of $C_2^{(+-)}$ so that, e.g., the first particle is always the one with the smaller momentum, we predict that

$$C_2^{(+,-)}(\mathbf{k}_1, \mathbf{k}_2) \neq C_2^{(-,+)}(\mathbf{k}_1, \mathbf{k}_2) \qquad \text{if} \quad Z_{res} \neq 0 \tag{17}$$

This can be determined in a straigth-forward manner in experiments, either directly or by determining the charge-asymmetry of the two-particle distribution functions,

$$R^{(+-)} = \frac{N^{(+-)}(\mathbf{k}_1, \mathbf{k}_2) - N^{(-+)}(\mathbf{k}_1, \mathbf{k}_2)}{N^{(+-)}(\mathbf{k}_1, \mathbf{k}_2) + N^{(-+)}(\mathbf{k}_1, \mathbf{k}_2)} \tag{18}$$

Proper 3-body Coulomb calculations to estimate the magnitude of this effect can be performed with the help of the ansatz (7).

4) Essentially, the method of *3)* works also for the Coulomb correction due to a net residual charge for unlike-particle correlation functions. The study of these correlation functions was proposed recently by Lednicky and collaborators to access the temporal sequence of the particle emission[25]. This kind of physical information is not directly accessible to like-particle correlation measurements, and may provide a unique information on the dynamics of particle production. Hence it is of great importance to determine experimentally the value of the charge asymmetry parameter of the correlation functions, Eq. (18). If

this parameter is non-vanishing, three-body Coulomb-wave calculations have to be applied for unlike-particle correlations functions before determining precisely the temporal sequence of particle emission from high-energy heavy ion experiments.

5) In particle physics, Andersson and Ringnér recently suggested to utilize the three-particle intensity correlation function $C_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ as a test of the hadronization mechanism to see the longitudinal stretching of the string field in the decay of quark–anti-quark jets[12]. The method to estimate the Coulomb effects imbedded in this problem is outlined in the present paper.

Although this list can be continued we stop here because the above discussion is sufficient to show that the presented results are rather useful for a broad range of physical problems in high-energy particle and heavy ion physics.

4 Summary

On the basis of an explicit, analytically given form of the three-body Coulomb wave function that is correct in a large (asymptotic) region of three-body configuration space, we have developed a new method to systematically correct for explicit three-body Coulomb effects which is applicable to data analysis in a broad range of measurements in high-energy physics. The Riverside approximation has been established as a limiting case for vanishing source sizes.

Specifically, we have worked out our approach for a system of three charged pions. There, we have numerically estimated that the Riverside approximation is not precise enough to determine the magnitude on the 5 -10 % level of the three-body Coulomb correction factor in the correlation function, if the characteristic HBT radius parameter is 5 - 10 fm, which is the range of interest in high-energy heavy ion physics.

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